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Mathematical Modelling and Simulation of the Evolution of Phosphorus in a Bilayer Lake

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***Abstract* – This paper analyses the mathematical model of the evolution of phosphorus in a bilayer lake based on the kinetic equations which describe this process. The mathematical modelling was developed using Scilab-Xcos and allows an over time visualization of the water levels evolution with the following parameters: soluble phosphorus (Ps) insoluble phosphorus (Pi) and total phosphorus (Pt) for the two layers of the lake, namely the epilimnion and the hypolimnion. Furthermore, this work presents a simulation of the above mentioned parameters.**

Keywords* – *lake, modelling, phosphorus, Scilab-Xcos, simulation

Nomenclature

P_{se} – *soluble phosphorus fraction of in epilimnion [mg/m^3];*
P_{ie} – *insoluble phosphorus fraction of in epilimnion [mg/m^3];*
P_{sh} – *soluble phosphorus fraction of in hypolimnion [mg/m^3];*
P_{ih} – *insoluble phosphorus fraction of in hypolimnion [mg/m^3];*
P_{te} – *total phosphorus in epilimnion [mg/m^3];*
P_{th} – *total phosphorus in hypolimnion [mg/m^3];*
A_t – *surface thermocline [m^2];*
V_e – *volume of water in epilimnion [m^3];*
V_h – *volume of water in hypolimnion [m^3];*
Q – *flow of water carried [m^3/day];*
W_{se} – *intake of soluble phosphorus in epilimnion [mg/day];*
W_{ie} – *intake of insoluble phosphorus in epilimnion [mg/ day];*
W_{sh} – *intake of soluble phosphorus in hypolimnion [mg/ day];*
W_{ih} – *intake of insoluble phosphorus in hypolimnion [mg/ day];*
k_{ce} – *consumption constant in epilimnion [day^{-1}];*
k_{te} – *transformation constant in epilimnion [day^{-1}];*
k_{th} – *transformation constant in hypolimnion [day^{-1}];*
v_e – *sedimentation rate in the epilimnion [m/ day];*
v_h – *sedimentation rate in the hypolimnion [m/ day];*
v_t – *transfer coefficient by thermocline [m/ day].*

1. INTRODUCTION

In order to illustrate a more detailed modelling of the nutrient transformation processes which take place in a lake, phosphorus is considered as a representative element with two distinct fractions: soluble phosphorus noted Ps and respectively insoluble noted Pi.

Phosphorus is an essential limiting factor for the eutrophication in a lake.

We know that in reality, during summer time, lakes go through a thermal stratification which has been fairly well exemplified in Figure 1. The surface layer is called epilimnion, contains hot water, is well lit and allows algal photosynthesis to convert dissolved nutrients into particulate organic matter. The deepest layer called hypolimnion contains low lighted cold waters and is not favourable for algal activity. The two areas are separated by metalimnion, an area of limited thickness but with a strong temperature gradient, called thermocline which significantly reduces vertical mixing.

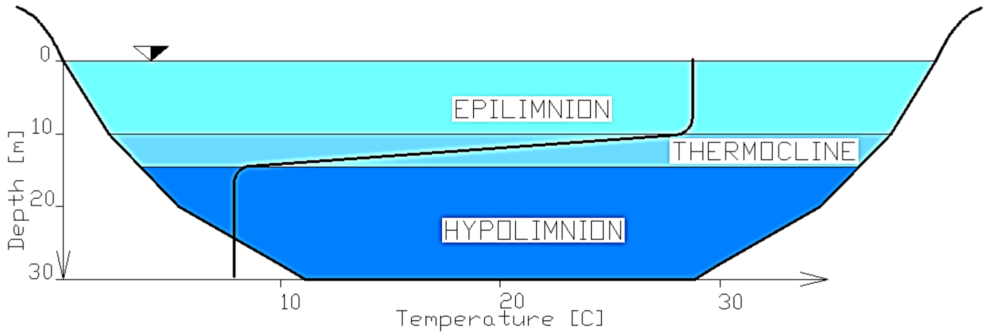

Fig. 1 Thermal stratification of the lake water during summer time

In autumn, water temperature decreases, the thermocline disappears and water masses form a circulation which brings nutrients, sediments, and gas from the bottom of the lake to the surface. In winter, the lake presents an inverse thermal stratification, with colder waters unsuitable for biomass activity on the surface.

During spring time, water temperature is relatively homogeneous and an inner movement contributes to the oxygenation of the lake. As solar radiation intensifies, there emerges the algal activity of the species which are more resistant to low temperatures (diatoms).

Taking into account these observations, the body of water can be schematized into two overlapping layers (each of them having homogeneous properties and corresponding to the epilimnion and hypolimnion), separated by an interface that allows certain transfers of constituents. The year is discretised into two seasons, winter and summer, which have different velocity values regarding the evolutionary processes.

Figure 2 outlines the processes included in the model and involving the two phosphorus fractions taken into consideration [1].

Using Scilab-Xcos, this article presents the description and the mathematical modelling of the phosphorus evolution in a lake.

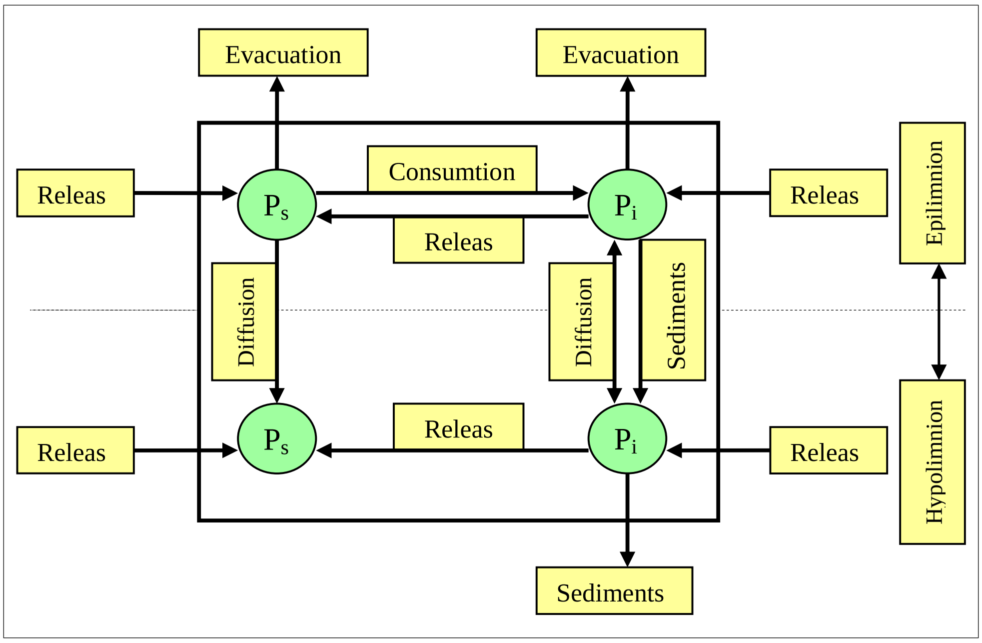

Fig. 2 Bilayer model of phosphorus processes

2. MATHEMATICAL MODEL OF THE EVOLUTION OF PHOSPHORUS IN A LAKE

By assigning lower indices e and h for sizes of the two layers, we can write the fallowing balance equations [1, 2]:

$$V_e * \frac{dP_{se}}{dt} = W_{se} - Q * P_{se} + v_t * A_t * (P_{sh} - P_{se}) - k_{ce} * V_e * P_{se} + + k_{te} * V_e * P_{ie} \tag{1}$$

$$V_e * \frac{dP_{ie}}{dt} = W_{ie} - Q * P_{ie} + v_t * A_t * (P_{ih} - P_{ie}) + k_{ce} * V_e * P_{se} - - k_{te} * P_{ie} - v_e * A_t * P_{ie} \tag{2}$$

$$V_h * \frac{dP_{sh}}{dt} = W_{sh} + v_t * A_t * (P_{se} - P_{sh}) + k_{th} * V_h * P_{ih} \tag{3}$$

$$V_h * \frac{dP_{ih}}{dt} = W_{ih} + v_t * A_t * (P_{ie} - P_{ih}) - k_{th} * V_h * P_{ie} - v_h * A_t * P_{ih} \tag{4}$$

$$P_{te} = P_{se} + P_{ie} \quad (5)$$

$$P_{th} = P_{sh} + P_{ih} \quad (6)$$

Based on the above equations [1, 2], we performed in Scilab-Xcos [5] the diagrams in Figures 3, 4 and 5 which can become the bases of various scenarios.

The program developed in Scilab-Xcos uses the Sundials / CVODE-BDF-NEWTON integration method and for the following example we used the time reference of 1 day for a simulation period of 365 days.

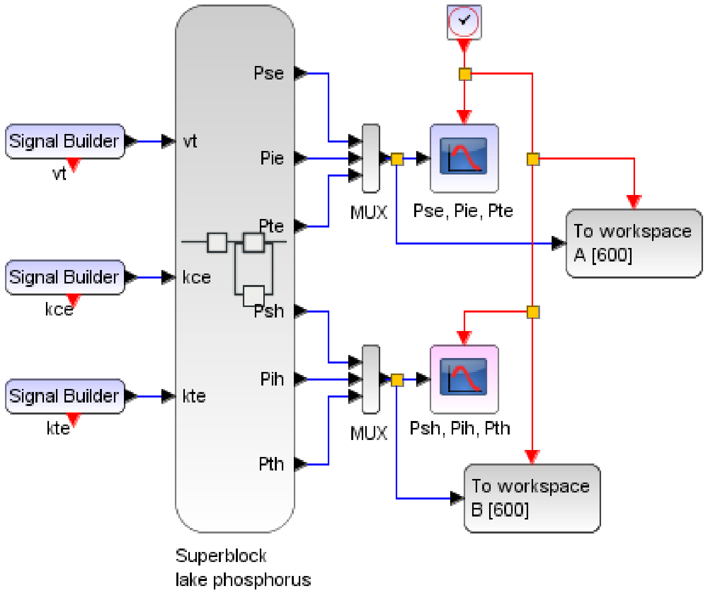

Fig. 3 Xcos simulation of the evolution of phosphorus in a lake

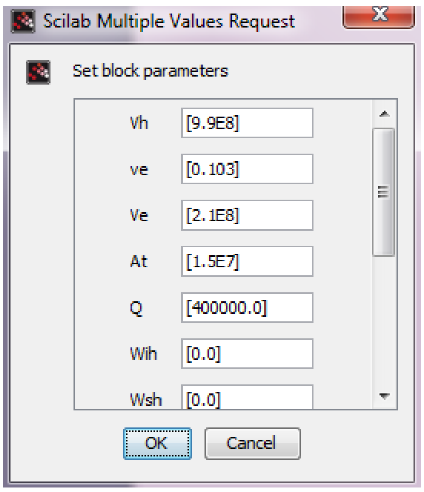

Fig. 4 Superblock parameters of the phosphorus in a lake, in Figure 3

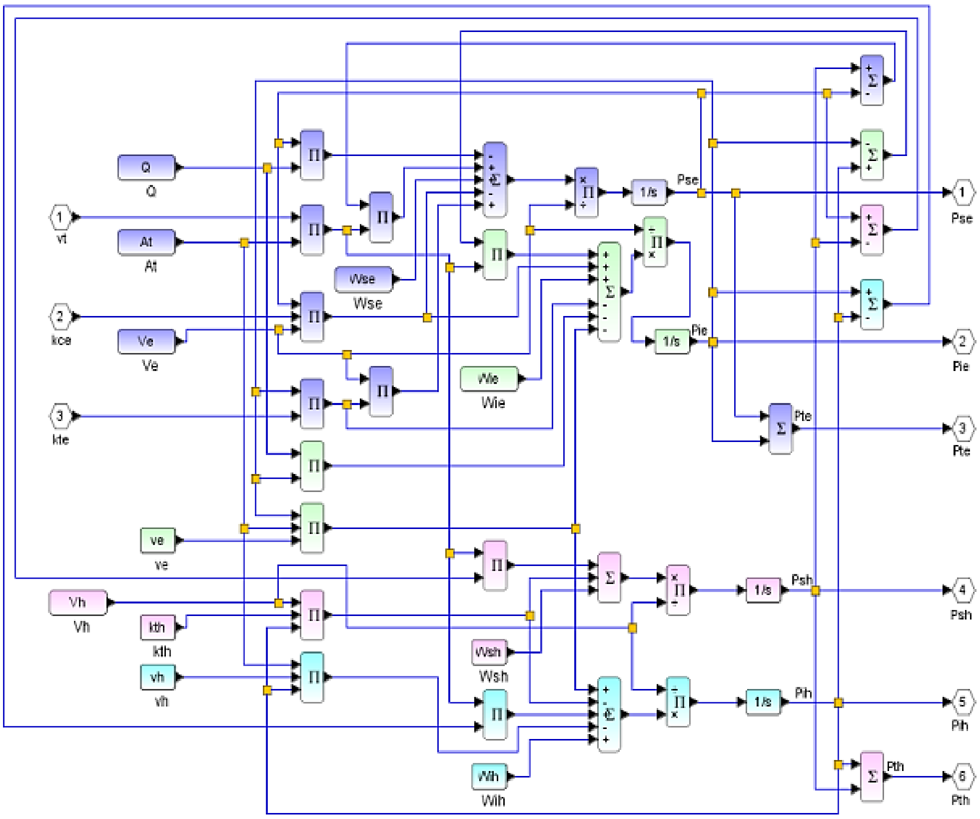

Fig. 5 Xcos Diagram of Superblock phosphorus parameters in a lake, in Figure 3

3. EXAMPLE OF SIMULATION

Initial data **Error! Reference source not found.1]:**

- The flow of water carried $Q=400000\ m^3/zi$;
- $V_e=2.1*10^8\ m^3$; $V_h=9.9*10^8\ m^3$; $Wse=7200000\ mg/zi$; $Wie=14400000\ mg/zi$; $W_{sh}=W_{ih}=0\ mg/zi$; $v_e=v_h=0.103\ m/zi$; $P_{se}=P_{ie}=P_{sh}=P_{ih}=11.2\ mg/m^3$;
- The time for which the simulation is done $t=365$ *days*;
- Transfer coefficient by thermocline v_t [m/day] evolves during the time period simulated according to the schedule below (Figure 6);

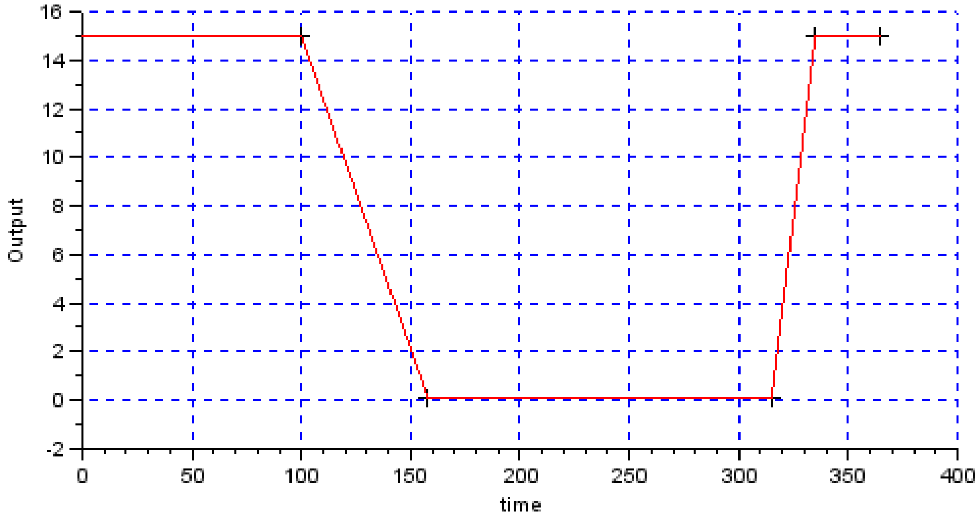

Fig. 6 Evolution of the transfer coefficient in thermocline (vt) during the simulated time

- Constant in epilimnion consumption kce [day^{-1}] has its simulated evolution according to the time schedule below (Figure 7);

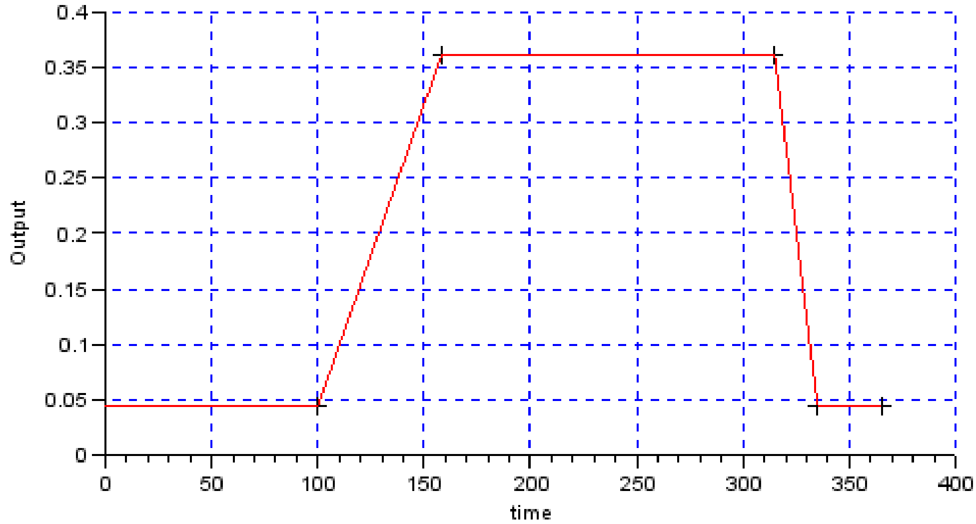

Fig. 7 Evolution of the consumption constant in epilimnion (kce) during the simulated time

- The transformation constant in epilimnion kte [day-1] evolves according to the time simulation scheduled below (Figure 8)

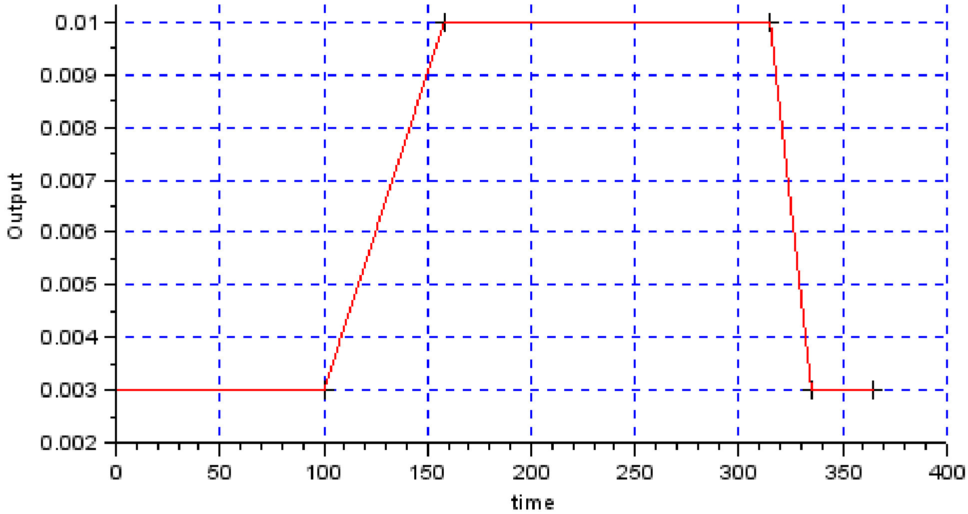

Fig. 8 Evolution of the transformation constant in epilimnion (kte) during the simulated time

Following the simulation, we can see graphically the way in which the fractions of soluble insoluble and total phosphorus have evolved in the two layers of water of a lake (epilimnion and hypolimnion) (Figures 9 and 10).

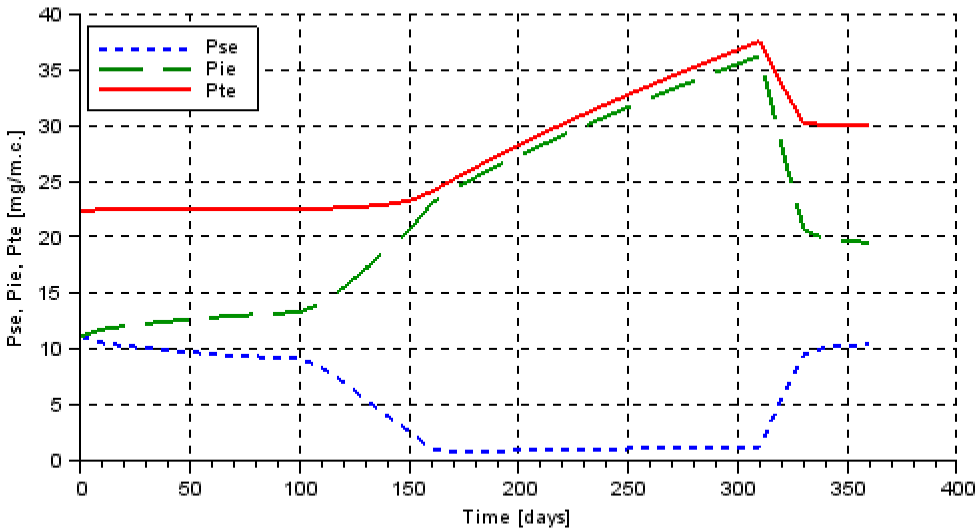

Fig. 9 Time evolution of the fraction and total phosphorus in the epilimnion layer of a lake

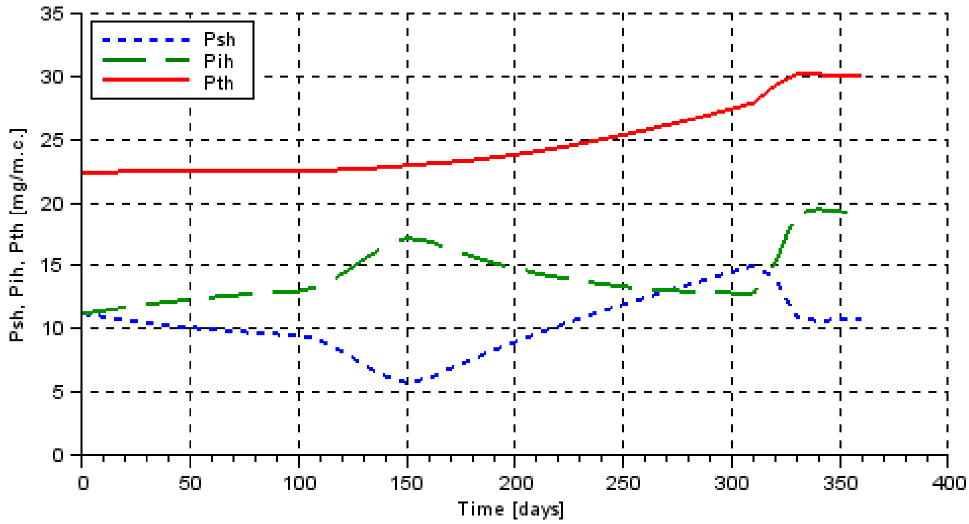

Fig. 10 Time evolution of the fraction and total phosphorus in hypolimnion layer of a lake

4. CONCLUSION

The calculation program (Figures 3, 4 and 5), developed by the author of this article in Scilab-Xcos, solves mathematical equations (1 ÷ 6) which describe the time evolution of the soluble, insoluble and total phosphorus fractions for the two layers of a lake (epilimnion and hypolimnion). It is useful for lake exploitation as it performs simulations which show the evolution of the above mentioned parameters over a certain time period and specific conditions for each simulated case, function of the variations of the determining parameters of these processes.

5. REFERENCES

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Note:

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